THE EFFECT WHICH THE THICKNESS OF VACUUM-LAMINATED INSULATION HAS ON ITS EFFECTIVE THERMAL CONDUCTIVITY

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We examine the heat-transfer mechanism in vacuum-laminated insulation and explain the relationship between the effective coefficient of thermal conductivity and the thickness of the insulation layer.

As demonstrated by experiments [1], the effective coefficient of thermal conductivity for vacuum-laminated insulation is not only a function of temperature, but also a function of thickness. To explain this phenomenon, let us initially examine the relationship between the effective coefficient of conductivity for an insulation packet and the effective coefficient of thermal conductivity for its separate segments. Here we will use the experimental data [1] for insulation that is arranged loosely and is made up of aluminum screens, with a thickness of 14 μm and SBR-M glass spacers with a thickness of 40 μm. We will divide the thickness of the insulation packet into n equal segments ($\delta_i = \delta/n$).

Since the specific heat flow in the steady-state regime is identical at any point on the specimen, the following equation is valid:

$$\lambda_{\text{eff } i}\frac{\Delta T_i}{\delta_i} = \lambda_{\text{eff}}\frac{T_1 - T_2}{\delta} = q = \text{const.} \tag{1}$$

Since $T_1 - T_2 = \sum_{i=1}^{n} \Delta T_i$, Eq. (1) can be transformed to

$$q\left(\frac{\delta}{n\lambda_{\text{eff }1}} + \frac{\delta}{n\lambda_{\text{eff }2}} + \cdots + \frac{\delta}{n\lambda_{\text{eff }n}}\right) = \frac{q\delta}{\lambda_{\text{eff}}}, \tag{2}$$

whence

$$\frac{1}{\lambda_{\text{eff}}} = \frac{1}{n}\left(\frac{1}{\lambda_{\text{eff }1}} + \frac{1}{\lambda_{\text{eff }2}} + \cdots + \frac{1}{\lambda_{\text{eff }n}}\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\lambda_{\text{eff } i}}. \tag{3}$$

Figure 1 shows the curves for the change in the effective coefficient $\lambda_{\text{eff } i}$ of thermal conductivity in the insulation segment, for specimens exhibiting thicknesses of 10, 21.5, and 40 mm; these curves have been plotted according to (1), where q and ΔT_i have been derived experimentally. The reduced specimen thickness x/δ has been plotted along the axis of abscissas.

It follows from the graph that: a) $\lambda_{\text{eff } i}$ varies sharply through the specimen thickness, and the maximum values are found in the middle zone; b) the absolute values of $\lambda_{\text{eff } i}$ increase in the various segments with an increase in the specimen thicknesses; c) the absolute values for specimens of identical thickness are greater for specimen boundary temperatures of 300-77°K.

As follows from (3) and Fig. 1 λ_{eff} as a function of insulation thickness is thus governed by the change in $\lambda_{\text{eff } i}$ in the segments.

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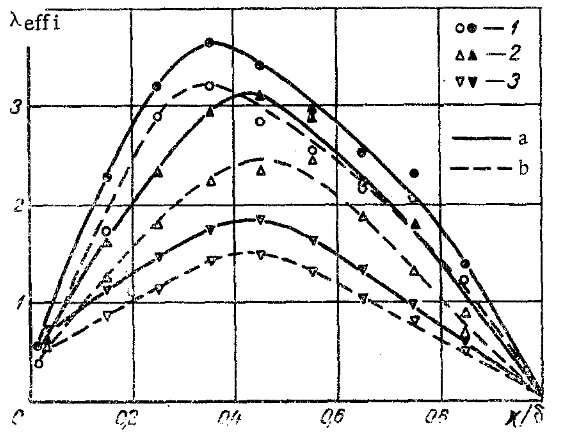

Fig. 1

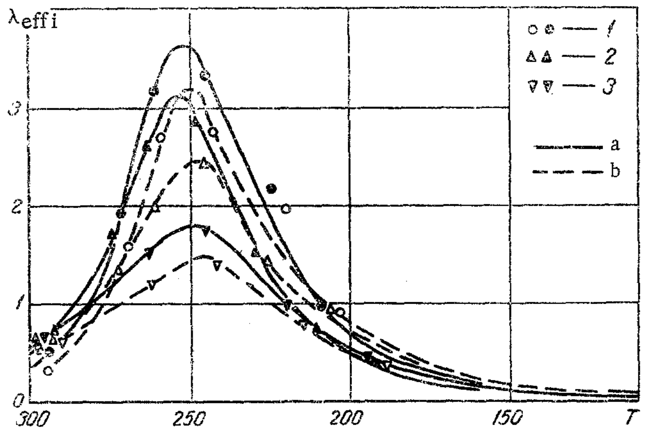

Fig. 2

Fig. 1. Change in the conductivity $\lambda_{eff\,i}$ of the insulation segment, μW/cm · deg for specimens of various thicknesses: 1) 40 mm; 2) 21.5 mm; 3) 10 mm; a) specimen temperature 300–77°K; b) 300–20.4°K.

Fig. 2. Change in conductivity $\lambda_{eff\,i}$, μW/cm · deg, in the insulation segments exhibiting identical temperatures at the boundaries, for specimens of various thicknesses: 1–3a and b) see Fig. 1.

It is most convenient to explain the nature of the function $\lambda_{eff\,i}(x)$ by examining the change in $\lambda_{eff\,i}$ in segments exhibiting identical temperatures at the boundaries of specimens of various thicknesses (Fig. 2). The plotting of $\lambda_{eff\,i}(T_i)$ was also accomplished on the basis of (1). As we can see from Fig. 2, in segments with identical boundary temperatures the absolute values of $\lambda_{eff\,i}$ increase with an increase in the thickness of the specimen. Let us explain this phenomenon.

The transfer of heat between the screens [aluminum foil] is accomplished by radiation, and by conduction through the solid and the residual gases. Thus we can write

$$\lambda_{eff\,i} = \lambda_{rad} + \lambda_s + \lambda_{eff\,g\,i} \tag{4}$$

Let us evaluate the effect of the components in the right-hand member of (4) and their contribution to the conductivity with a change in insulation thickness. In the specimen segments exhibiting identical temperature at the boundaries we have $\lambda_{rad} \approx$ const, since the screens have been fabricated from one material and their packing density is constant (28 screens/cm). This is also borne out by curves 3 (Fig. 4). Under identical conditions, in first approximation, we can also assume that $\lambda_s \approx$ const (see curves 2 in Fig. 4). Consequently, the function $\lambda_{eff\,i}(x)$ can be explained by a change in $\lambda_{eff\,g\,i}$.

Let us evaluate the residual pressure in the layers of the insulation and the nature of its variation with a change in specimen thickness (Fig. 3). The curves have been plotted according to the method described in [2] for the case of a minimum pressure of $2 \cdot 10^{-5}$–$5 \cdot 10^{-5}$ N/m^2 in the calorimeter bomb. Let us examine the relationship between $\lambda_{eff\,g\,i}$ and the residual pressure between the screens. The flow of heat between two adjacent screens, transmitted by the residual gases, can be written as

$$q_g = \frac{\lambda_{eff\,g\,i}(T_i - T_{i+1})}{\delta/N}. \tag{5}$$

On the other hand, assuming approximately that because of the substantial porosity of the glass spacer ($m > 0.9$) the gas molecules pass through it freely, without collision against any of the fibers, and because $Kn = (L/d) \geq 1$ (this condition is satisfied in our case), according to the kinetic theory of gases the transfer of heat by the residual gases between the screen [3] can be determined from the equation

$$q_g = \frac{\lambda_{g\,i}(T_i - T_{i+1})}{2\beta C} P, \tag{6}$$

where $\lambda_{g\,i}$ is the coefficient of thermal conductivity for the residual gas at the temperature $(T_i + T_{i+1})/2$:

$$\frac{C}{P} = L; \quad \beta = \frac{2-a}{a}\,\frac{2\mu}{\gamma+1}; \quad \mu = \frac{9\gamma - 5}{4}.$$

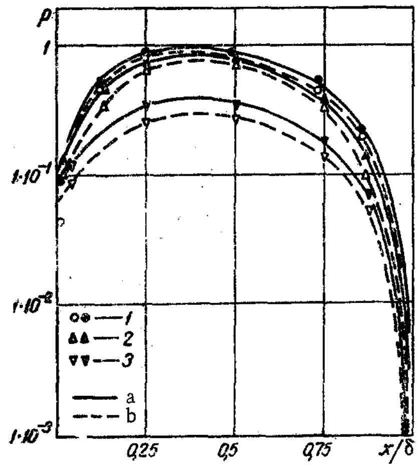

Fig. 3

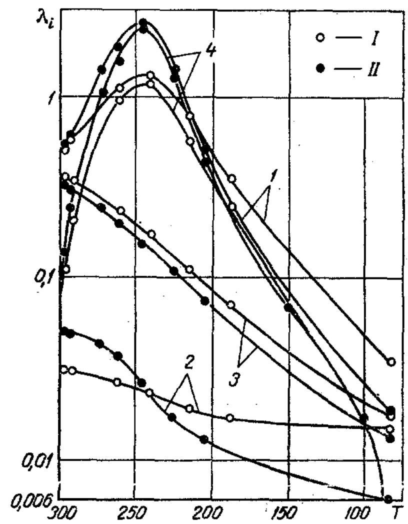

Fig. 4

Fig. 3. Distribution of pressure P for residual gases in the insulation layers, N/m^2, for specimens of various thicknesses: 1-3, a, and b) see Fig. 1.

Fig. 4. Overall conductivity λ_i, $\mu W/cm \cdot deg$, in the insulation segments with identical temperatures at the boundaries, and the components of the conductivity for the solid, the radiation, and the residual gases in the layers for specimens of the following thickness: I) 10 mm; II) 21.5 mm; 1) $\lambda_{eff\,i}$; 2) λ_s; 3) λ_{rad}; 4) $\lambda_{eff\,g\,i}$.

Equating Eqs. (5) and (6), we find that

$$\lambda_{eff\,g\,i} = \frac{\lambda_{g\,i}\delta}{2\beta CN} P. \tag{7}$$

Turning to any specimen thickness and assuming in all cases that T_i = const, and assuming that the change in T_{i+1} is insignificant (according to the experimental data), we can operate on the assumption that in (7) the quantity $\lambda_{g\,i}/2\beta C$ = const. Then $\lambda_{eff\,g\,i} = \lambda_{eff\,g\,i}(P)$, and consequently, $\lambda_{eff\,i} = \lambda_{eff\,i}(p)$. At the same time, we see from Fig. 3 that the absolute values of the pressure in the layers increase with an increase in specimen thickness, i.e., P = P(x). With this relationship, it follows from (7) that $\lambda_{eff\,g\,i}$ and, consequently, the values of $\lambda_{eff\,i}$ in (4) are functions of the thickness. The increase in the absolute values of the pressure in the insulation layers (with an increase in specimen thickness) is explained by the increase in the gas-release surface, whereas the evacuation conditions are impaired. Reference is made in [4] to the poor conditions of evacuation from the insulation and to the resulting difference in pressures between the layers and within the insulated space.

As we can see from Fig. 3, the greatest pressure values are found in the middle zones. Therefore, according to (7), in these zones $\lambda_{eff\,g\,i}$ is at its greatest and, consequently, $\lambda_{eff\,i}$ is at its maximum (see Figs. 1 and 2). It should be expected that with an increase in specimen thickness for identical boundary temperatures (provided that the conditions of a constant rate of evacuation from the calorimeter bomb is maintained) a pressure will be established within the middle zone of the specimens that is close to the constant pressure which is brought about by the dynamic equilibrium between the number of evacuated molecules and those being released from the material. This assumption is confirmed by the curves of Fig. 3. Since the effect of the extreme zones becomes insignificant in this case, $\lambda_{eff\,i}$ will tend toward some constant quantity.

Let us evaluate the contribution to the overall transfer of heat in the insulation layers by radiation and by conduction through the solid and through the residual gases; for this purpose we will use two specimens, with thicknesses of 10 and 21.5 mm, at boundary temperatures of 300-20.4°K. The tentative data for the components of Eq. (4) are given in Fig. 4.

Radiative heat transfer (curves 3) was determined from the equation

$$\lambda_{\mathrm{rad\ i}} = \frac{q_{\mathrm{rad\ i}}\,\delta/N}{T_i - T_{i+1}}, \tag{8}$$

where

$$q_{\mathrm{rad\ i}} = \varepsilon_{\mathrm{red\ i}}\,\sigma\,(T_i^4 - T_{i+1}^4).$$

The values of $\varepsilon_i(T)$ are taken from [5].

The conduction through the solid was determined in the following manner. The transfer of heat between the cold wall of the calorimeter exhibiting a temperature of 20.4°K and the adjacent screen, at a pressure of $2 \cdot 10^{-5}$ N/m^2 in the calorimeter bomb, is due exclusively to radiation and conduction through the spacer. Thus

$$\lambda_{\mathrm{s}} = \frac{(q - q_{\mathrm{rad\,n}})\,\delta/N}{T_n - T_2}. \tag{9}$$

Since the resulting values of $\lambda_{\mathrm{s}0}$ in our case are smaller by a factor of 10^2-10^3 than the thermal conductivity of the glass when T = 100°K [6], recalculated with provision for the density of the glass paper and the diameter of its elementary fibers, we can assume that $\lambda_{\mathrm{s}0}$ is defined exclusively by contact conduction. In the existing formulas for contact conduction in fiberglas materials (for example, Eq. (58) on p. 34 of [7]) the authors assumed a model in which the area of the contact spots varies with a change in the applied load as $\sqrt[3]{p}$, while the number of these contact points remains constant. Such a model is quite valid for great loads. However, in our case, with the specific load ranging from 0.05 to 0.36 g/cm^2 (with consideration of specimen weight), there is an increase in the number of contacts with application of the load, and the change in contact conduction, in first approximation, can be assume to be linearly dependent on the load [8]. Our experimental data differ from the theoretical data obtained from formula (58) of [7] by factors of 200-400.

To determine the function $\lambda_{\mathrm{s}0}$ we performed a number of experiments for three values of the specific load on the specimen: p = 0 is the loose packing; p = 0.05 and 0.11 g/cm^2. From the resulting magnitudes of of the heat flow and the temperature differences between the cold wall and the screen in contact with the wall, using formula (9), we determined the contact conductivity which is expressed by the relation

$$\lambda_{\mathrm{s}\,0} = 0.36p. \tag{10}$$

Figure 4 shows the curves for the change in contact conductivity through the thickness of the specimen (curves 2), plotted according to (10), where the specific load p was determined with consideration of the initial compression of the specimen and with consideration of the weight of the insulation layer.

The conductivity through the gas (curves 4) was determined from the difference

$$\lambda_{\mathrm{eff\ g\ i}} = \lambda_{\mathrm{eff\ i}} - (\lambda_{\mathrm{rad\ i}} + \lambda_{\mathrm{s\ i}}).$$

Table 1 shows the mean integral values of the conductivities for radiation, and for conduction through the solid and through the residual gas.

Analytically, let us express λ_{eff} as a function of the insulation thickness and the pressure of the residual gas in the layers on the basis of the above-cited analysis of the heat-transfer mechanism. From the theory of heat transfer for multilayer materials, in the case of $\lambda = \lambda(T)$, the heat flow in a steady-state regime is written as follows:

$$q = \frac{\lambda_{\mathrm{eff}}(T_1 - T_2)}{\delta}, \tag{11}$$

where

$$\lambda_{\mathrm{eff}} = \frac{1}{T_1 - T_2}\int_{T_1}^{T_2} \lambda(T)\,dT. \tag{12}$$

It follows from a comparison of the experimental data (Figs. 1-4 and Table 1) and their analysis that λ_{eff} is a weak function of temperature and is determined primarily by the residual pressure in the insulation layers. This can be seen from comparison of the pressure and $\lambda_{\mathrm{eff\,i}}$ at the boundaries of the specimen

TABLE 1. Contribution to the Overall Transfer of Heat by Radiation, and Conduction through the Solid and through the Residual Gases in the Insulation Layers

δ, mm	λ_{eff}		$\lambda_{eff\ g}$		λ_{rad}		λ_s	
	I	II	I	II	I	II	I	II
21,5	0,51	100	0,42	83	0,073	14	0,014	3
10	0,4	100	0,3	74	0,088	22	0,016	4

and in the middle zone. The function $\lambda = \lambda(T)$ can therefore be neglected and in first approximation we can assume that $\lambda \approx \lambda_0 + \lambda(P)$, where $\lambda_0 = \text{const}$ is the average value of the effective coefficient of thermal conductivity for the given range of temperatures in the case in which there is no conduction through the gas.

As we can see from Figs. 2 and 3, the distribution of the residual-gas pressure in the insulation layers, in first approximation, is independent of temperature, but it is a function of the coordinates, i.e., $P = P(x)$. We can then write that

$$\lambda = \lambda_0 + \lambda(x). \tag{13}$$

Therefore, for our case, the heat-conduction equation assumes the form

$$-[\lambda_0 + \lambda(x)]\frac{dT}{dx} = q = \text{const}. \tag{14}$$

Let us integrate Eq. (14) over the entire insulation thickness δ

$$-\int_{T_1}^{T_2} dT = q\int_0^{\delta}\frac{dx}{\lambda_0 + \lambda(x)}. \tag{15}$$

As a result of the integration we have

$$q = \frac{T_1 - T_2}{\int_0^{\delta}\frac{dx}{\lambda_0 + \lambda(x)}}. \tag{16}$$

Let us introduce λ_{eff} in the following manner:

$$\lambda_{eff} = \frac{\delta}{\int_0^{\delta}\frac{dx}{\lambda_0 + \lambda(x)}}. \tag{17}$$

Let us transform Eq. (17). Since $P = P(x)$, we can express x in the form of the function $x = x(P)$. Let us find the total differential of this expression, bearing in mind that the function $\lambda = \lambda(T)$ can be neglected:

$$dx = \frac{\partial x}{\partial P}dP = \frac{dP}{\frac{\partial P}{\partial x}}. \tag{18}$$

After substitution into (17), λ_{eff} finally assumes the form

$$\lambda_{eff} = \frac{\delta}{\int_{P_1}^{P_2}\frac{dP}{\frac{\partial P}{\partial x}[\lambda_0 + \lambda(P)]}}. \tag{19}$$

Finally, we can draw the following conclusions: a) in vacuum-laminated insulation we have the function $\lambda_{eff}(\delta)$, which is governed by the presence of the residual gas in the layers. With an increase in thickness the absolute values of the pressures in the layers increase as a result of impairment of the evacuation conditions; b) even in the case in which the pressure on the specimen is lower than $1 \cdot 10^{-3}$ N/m^2, the effective coefficient of thermal conductivity for vacuum-laminated insulation should be treated as a function of temperature and as a function of the residual pressure in the insulation layers; c) in vacuum-laminated insulation based on aluminum foil and SBR-M glass paper, with the layers freely packed, the principal contribution to the heat transfer is made by the residual gases and by radiation (see Table 1). In this case, the transfer of heat through the solid amounts to no more than 5%.

NOTATION

λ_{eff}	is the effective coefficient of thermal conductivity;
q	is the specific heat flow;
δ	is the specimen thickness;
x	is the instantaneous coordinate;
N	is the number of screens;
n	is the number of segments;
i	is the sequential number of each segment;
T	is the temperature;
T_1	is the temperature of the warm wall;
T_2	is the temperature of the cold wall;
Kn	is the Knudsen number;
L	is the mean free path of the gas molecules;
d	is the fiber diameter;
P	is the gas pressure;
a	is the accommodation factor;
$\gamma = C_p/C_v$	is the ratio of isobaric and isochronic heat capacities;
p	is the specific load;
σ	is the Stefan–Boltzmann constant.

LITERATURE CITED

1. R. S. Mikhal'chenko, A. G. Gerzhin, V. T. Arkhipov, and N. P. Pershin, Inzh. Fiz. Zh., 15, No. 3 (1968).
2. R. S. Mikhal'chenko, A. G. Gerzhin, V. T. Arkhipov, N. P. Pershin, and L. V. Klipach, Inzh. Fiz. Zh., 14, No. 1 (1968).
3. S. Dushman, Scientific Foundations of Vacuum Engineering [Russian translation], Mir, Moscow (1964).
4. J. P. Debbrane, Le Vide, No. 119, 323 (1965).
5. R. S. Mikhal'chenko, A. G. Gerzhin, and N. P. Pershin, Inzh. Fiz. Zh., 14, No. 3 (1968).
6. M. P. Malkov, I. B. Danilov, A. G. Zel'dovich, and A. B. Fradkov [in Russian], Gosenergoizdat, Moscow–Leningrad (1963).
7. M. G. Kaganer, Thermal Insulation in Low-Temperature Engineering [in Russian], Mashinostroenie, Moscow (1966).
8. P. E. Glaser, I. A. Black, and P. R. Doherty, Mechanical Eng., 8, 23 (1965).